

Cornell University

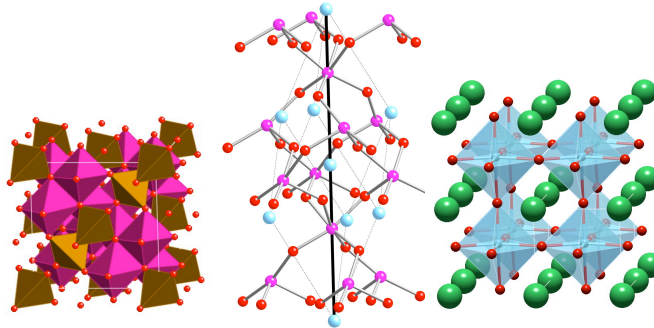
"I would found an institution where any person can find instruction in any study."  
– Ezra Cornell, 1868

Basic Training 2009– Lecture 01

# Competing Ferroic Orders

The magnetoelectric effect

Craig J. Fennie  
School of Applied and Engineering Physics  
fennie@cornell.edu



Basic Training in Condensed  
Matter Theory 2009

## Module Outline (Extremely tentative)

### 1. Overview and Background

- Ferro ordering, the magnetoelectric effect
- Complex oxides basics: Types of insulators (i.e., ZSA classifications), Coordination chemistry

### 2. Structure and Ferroelectricity

- Basics of space groups
- Soft mode theory, lattice dynamics, group theoretic methods
- Competing lattice instabilities
- microscopic mechanisms, improper FE
- Modern theory of polarization (Berry Phase)

### 3. Magnetism

- Basics, exchange interactions, superexchange, Dzyaloshinskii-Moria
- How spins couple to the lattice! Phenomenology and microscopies (spin-phonon, spin-lattice, etc)
- Competing magnetic orders

### 4. Lets start putting things together

- Phase competition: magnetic and polar orders and colossal magnetoelectric responses
- Magnetic order induced ferroelectricity
- Ferroelectric induced ferromagnetism, switching magnetism  $180^\circ$  with an electric field aren't these forbidden by some symmetry? NO!

### 5. Finish up loose ends and recent papers I wish I understood better

- Toroidal moments



Cornell University  
School of Applied and Engineering Physics

Basic Training 2009– Lecture 01

2

# Ab initio Yes a play on what I do.

➤ What does my title mean?

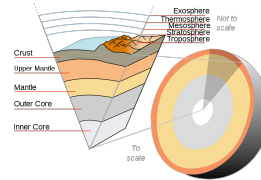
“ferro” derived from the Latin “ferrum” → Iron

- Iron is the sixth most abundant element in the Universe

- While it makes up about 5% of the Earth's crust, the Earth's core is believed to consist largely of an iron-nickel alloy constituting 35% of the mass of the Earth as a whole.

- Iron is consequently the most abundant element on Earth, but only the fourth most abundant element in the Earth's crust.

- Most of the iron in the crust is found combined with oxygen as iron oxide minerals such as **hematite** and magnetite.

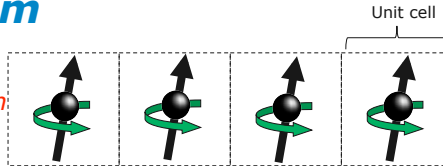


<http://en.wikipedia.org/wiki/Iron>

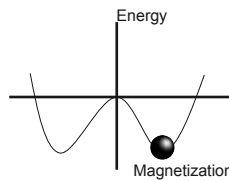
# Ferromagnetism

Ordering of spins → spontaneous magnetization:

$$M \equiv \sum_i \langle S_i \rangle \neq 0$$

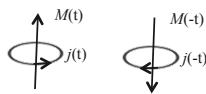


Spontaneous time-reversal, **R**, symmetry breaking

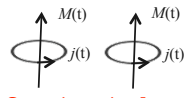


$$F(M) = \alpha M^2 + \beta M^4 - MH$$

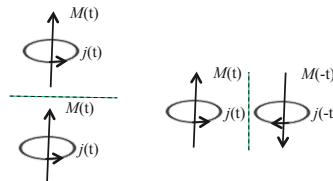
Symmetry properties  $M(-t) = -M(t)$



Time inversion **R**  $t \rightarrow -t$



Space inversion **I**  $r \rightarrow -r$   
 $M$  and  $H$  are Axial vectors



Mirror reflection **m**

# Ferroelectricity

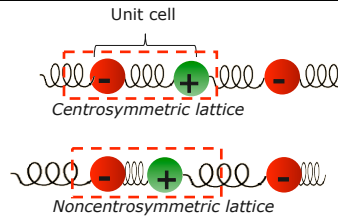
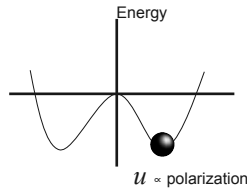
Spontaneous polarization  $\Rightarrow$  Dipole moment per unit volume

## Ordering of polar mode

in many cases, this turns out to be a lattice mode

$$P_\alpha \approx \sum_i Z_{\alpha,\beta;i}^* \langle u_{\beta;i} \rangle \neq 0$$

Spontaneous space-inversion,  $I$ , symmetry breaking

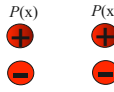


$$F(u) = \alpha u^2 + \beta u^4 - Z^* \cdot u \cdot E$$

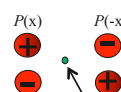
OR

$$F(P) = \alpha P^2 + \beta P^4 - PE$$

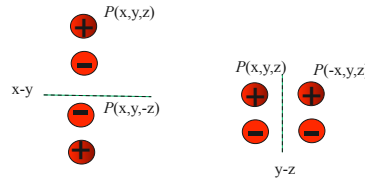
Symmetry properties  $P(-x) = -P(x)$



Time inversion  $R$   $t \rightarrow -t$



Space inversion  $I$   $r \rightarrow -r$   
 $P$  and  $E$  are Polar vectors



Mirror reflection  $m$



Cornell University  
School of Applied and Engineering Physics

Basic Training 2009- Lecture 01

5

# AntiFerromagnetism

$q \neq 0$ , Ordering of spins

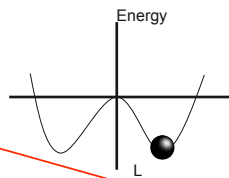
$$M \equiv \sum_i \langle S_i \rangle = 0$$

e.g., two sublattice spin system

$$L = S_1 - S_2$$

Symmetry properties

$$L(-t) = -L(t) ; L(-x) = \pm L(x)$$



$$F(M) = \alpha L^2 + \beta L^4 - LH_q$$

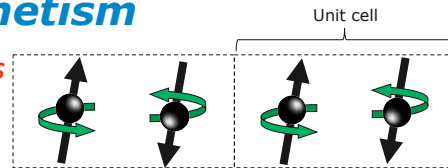
Does the AFM ordering of spins spontaneously break time-reversal,  $R$ , symmetry breaking?

First, what does the question mean?

Does the point group of the space group contain  $R$ ?

Depends on details of crystallographic structure!

Cant ignore structure!!  
i.e., LATTICE + BASIS  
(don't just pick your favorite sublattice and ignore the rest!!!)



Cornell University  
School of Applied and Engineering Physics

Basic Training 2009- Lecture 01

6

# AntiFerromagnetism

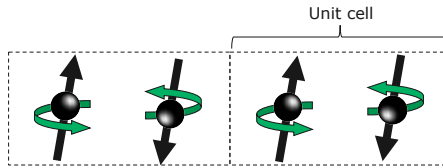
Does the AFM ordering of spins spontaneously break time-reversal,  $R$ , symmetry breaking ?

Symmetry properties  $L(-t) = -L(t)$  ;  $L(-x) = \pm L(x)$

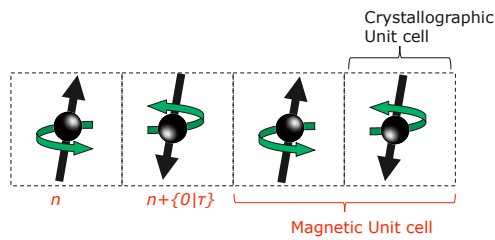
e.g., two sublattice spin system (in the ordered phase)

$$L = S_1 - S_2$$

Example 1: two spins sit in the same unit cell in the paramagnetic phase, Yes!  $R$  is not a symmetry element.



Example 2: two spins sit in different unit cells in the paramagnetic phase, No! the space group contains  $\{R|\tau\} \Rightarrow$  the point group,  $\tau \rightarrow 0$  contains  $\{R|0\}$ .



# Ab initio

Yes a play on what I do.

➤ What does my title mean?

Def: Linear magnetoelectric effect

Landau and Lifshitz, "Electrodynamics of continuous media"  
Dzyaloshinskii, JETP 1957; Astrov JETP 1960  $\rightarrow$   $\text{Cr}_2\text{O}_3$   
Hans Schmid, Geneva; Smolenskii USSR

Requirements: 1) broken space-inversion symmetry  
2) broken time-reversal symmetry

$$\mathcal{F} = \mathcal{F}_0 + P_s E + M_s H + \epsilon E^2 + \mu H^2 + \gamma E H$$

$$P = \left. \frac{\partial \mathcal{F}}{\partial E} \right|_{E=0} = P_s + \gamma H$$

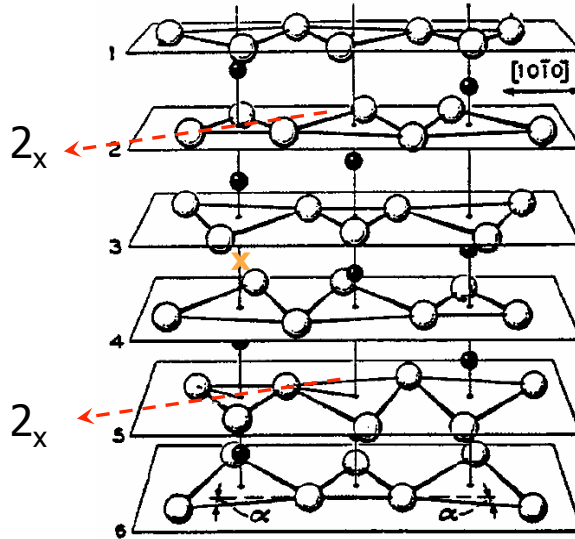
$$M = \left. \frac{\partial \mathcal{F}}{\partial H} \right|_{H=0} = M_s + \gamma E$$

$\gamma \equiv$  linear magnetoelectric coefficient



From Max Mostovoy

## Crystal structure of $\text{Cr}_2\text{O}_3$



From Max Mostovoy

## $\text{Cr}_2\text{O}_3$

AFM order parameter  $T_N = 306\text{K}$

$$\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2 + \mathbf{M}_3 - \mathbf{M}_4 \quad L_z \neq 0$$

symmetries of paramagnetic phase

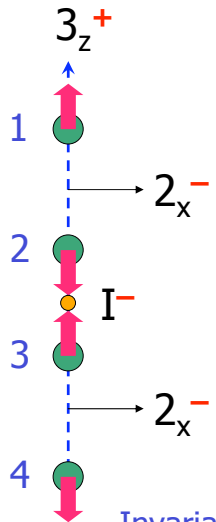
	$I$	$2_x$	$3_z$
$L_z$	-	+	+
$E_z$	-	-	+
$H_z$	+	-	+

point group

$\bar{3}m$

$1, 3(2_\perp), \pm 3_z$

$\bar{1}, 3(m_\perp), \pm \bar{3}_z$



$$\lambda L_z E_z H_z = \alpha_\parallel E_z H_z$$

Invariants:

$$L_z (E_x H_x + E_y H_y)$$

$$\alpha_\parallel, \alpha_\perp \propto L_z$$

From Max Mostovoy



magnetic point group  $\bar{3}'m'$

Symmetries of low-T phase:  $1, 3(2_\perp), \pm 3_z, \bar{1}', 3(m'_\perp), \pm \bar{3}'_z$

	$I'$	$2_x$	$3_z$
$\begin{pmatrix} E_x \\ E_y \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$R_{2\pi/3}$
$E_z$	-1	-1	+1
$\begin{pmatrix} H_x \\ H_y \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$R_{2\pi/3}$
$H_z$	-1	-1	+1

Inversion combined with time reversal

$$I' = IT$$

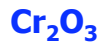
120°-rotation

$$R_{2\pi/3} = \frac{1}{2} \begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$$

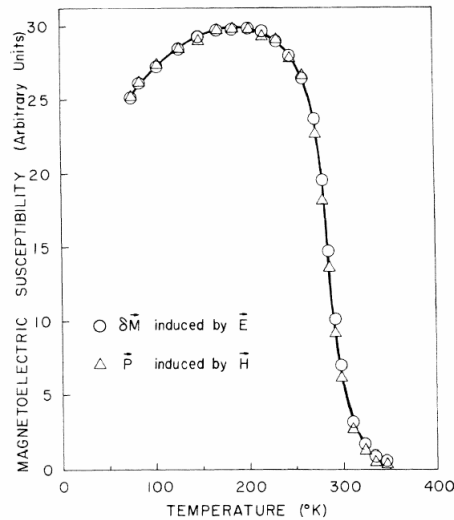
Invariants:  $F_{me} = -\alpha_{\parallel} E_z H_z - \alpha_{\perp} (E_x H_x + E_y H_y)$

From Max Mostovoy

## Linear magnetoelectric effect



*I. E. Dzyaloshinskii JETP 10 628 (1959),  
D. N. Astrov, JETP 11 708 (1960)*



$$P = \chi_e E + \alpha H$$

$$M = \alpha E + \chi_m H$$

*G.T. Rado PRL 13 335 (1964)*

## *Next time*

### ME revisited, and basic oxide physics

- [ME effect revisited: Toroidal moments](#)
- Complex oxides basics: Types of insulators (i.e., ZSA classifications), Coordination chemistry

